2.1 (a) A hiker absorbs 1.00 L of water. It evaporates at 20 ºC. How much heat is required for this process? (b) What would the temperature drop for the hiker be (assuming the clothes and hiker have a mass of 60 kg and a heat capacity of water)? (c) How many grams of sucrose is needed to be metabolized to account for this amount of heat?

a) Evaporation is a phase change (liq → gas), so we use the latent heat of vaporization of water at 20 ºC from Table 2.2. This is a process at constant pressure, so

\[ q_P = \Delta H = mL_v \]

where \( L_v \) is the (latent) heat of vaporization and \( m \) is mass.

The mass \( m = (1 \text{ L})(0.9982 \text{ g/cm}^3)(1000 \text{ cm}^3/\text{L}) = 998.2 \text{ g} = 0.9982 \text{ kg} \)

Thus the required heat is \( q_P = mL_v = (0.9982 \text{ kg})(2447 \text{ kJ/kg}) = 2443 \text{ kJ} \)

b) This is a problem in heat capacity, so

\[ q_P = C_p \Delta T = mC_p \Delta T \]

where \( C_p \) is the heat capacity of the whole system and the value with the bar over it is the specific heat capacity, that is the per unit mass or per mole value.

Rearranging this equation leaves

\[ \Delta T = \frac{q_P}{mC_p} = -\frac{2443 \text{ kJ}}{(60 \text{ kg})(4.184 \text{ kJ/kg} \cdot \text{K})} = -9.73K \]

where the minus sign for \( q_P \) comes from the fact that heat is being removed from the system.

c) This is a problem about the enthalpy change in a chemical reaction. We use

\[ \Delta H = \sum_{\text{prod}} \Delta H_f - \sum_{\text{react}} \Delta H_f \]

where the sums are of the enthalpies of formation of products and reactants. For the sucrose reaction, taking into account the numbers of moles of products and reactants, the combustion of one mole of sucrose has an enthalpy change of

\[ \Delta H = (1 \text{ mol})(-241.8 \text{ kJ/mol}) + (12 \text{ mol})(-393.5 \text{ kJ/mol}) - (1 \text{ mol})(-2222 \text{ kJ/mol}) - (12 \text{ mol})(0 \text{ kJ/mol}) = -5160 \text{ kJ} \]

This is an exothermic reaction. To produce 2442 kJ, the hiker would need to burn \((2442 \text{ kJ})/(5160 \text{ kJ/mol}) = 0.47 \text{ mol} \) of sucrose. Given that the molar mass of sucrose is 342.3 g/mol, this would amount to a mass of

\[ m = (#\text{mol}) \times \text{M.W.} = (0.47 \text{ mol})(342.3 \text{ g/mol}) = 162 \text{ g} \]
2.4. (a) As heat is added to ice at constant P, T increases until the ice melts. After melting, T continues to increase. Complete the plots for q vs. T and $C_P$ vs. T. (b) As heat is added to a weak ionic solution, q is linear with T. If DNA is added to the buffer, the DNA strands separate. Draw the q vs. T and $C_P$ vs. T curves.

a) Until the ice melts, the heat added will be proportional to the T of the ice. This is because $C_P$ is (very nearly) a constant and $q = C_P \Delta T$. When the ice melts, the T will arrest at the melting T (0 ºC) until the sample is all liquid. Then the relationship will be linear again, with a larger slope since $C_P$ of water is larger than $C_P$ of ice. That is enough to describe the q vs. T plot. Note that $C_P = dq/dT = \text{slope of the q vs. T plot}$. That is enough to describe the $C_P$ vs. T plot:

![Graphs](image)

b) There is a phase change as hydrogen and other noncovalent bonds are broken in the DNA. This occurs over a range of T since not every bond has the same strength. This means that the heat required (beyond heating the solution) will increase while the DNA is unwinding. The phase transition will be broad, not sharp as in a). The $C_P$ vs. T plot is the derivative of the q vs. T plot, so it will have a broad peak centered at the melting T. We could measure the heat of melting of DNA by integrating the $C_P$ vs. T curve over the T range where the phase transitions occurs, and subtracting the same integral for pure buffer with no DNA: In the plots, the DNA solution is shown as solid, the plain buffer alone as dashed.

![Graphs](image)
2.7 One mol of an ideal gas at 27 °C and 1 atm is heated and expands reversibly at constant P to T = 327 °C. Given that $C_V = 20.8 \text{ J/K·mol}$, calculate (a) the work done on the gas, (b) $\Delta E$ and $\Delta H$ and (c) the heat absorbed.

a) The expansion is at constant pressure, so $w = -P_{\text{ext}} \Delta V$. We need to compute $V_i$ and $V_f$.

$$V_i = \frac{nRT_i}{P} = \left(1 \text{ mol}\right) \frac{0.08205 \text{ L·atm/mol·K}}{1 \text{ atm}} \frac{300 \text{ K}}{1 \text{ atm}} = 24.6 \text{ L}$$

$$V_f = \frac{nRT_f}{P} = \left(1 \text{ mol}\right) \frac{0.08205 \text{ L·atm/mol·K}}{1 \text{ atm}} \frac{600 \text{ K}}{1 \text{ atm}} = 49.2 \text{ L}$$

So $\Delta V = V_f - V_i = 49.2 \text{ L} - 24.6 \text{ L} = 24.6 \text{ L}$

and thus $w = -P_{\text{ext}} \Delta V = -(1 \text{ atm})(24.6 \text{ L})(101.3 \text{ J/L·atm}) = -2.49 \text{ kJ}$, i.e. work is done by the gas as it expands.

b) Recall that, for an ideal gas, $\Delta E = q_P + w_P = C_P \Delta T + w_p$ and $C_p = C_V + nR$ (see eq. 2.31b and the discussion above it). From this we can calculate $C_p$ and, with the answer to a) the value of $\Delta E$. If you look at p. 43, eq. 2.32, you see that this is all combined, with some algebra, into a simpler relationship:

$$\Delta E = C_V \Delta T = (1 \text{ mol})(20.8 \text{ J/K·mol})(300 \text{ K}) = 6.24 \text{ kJ}.$$ 

Now, moving to $\Delta H$, we recall that $\Delta H = \Delta E + \Delta(PV) = \Delta H = \Delta E + P \Delta V$ at constant P.

This is easy to compute: $\Delta H = 6.24 \text{ kJ} + 2.49 \text{ kJ} = 8.73 \text{ kJ}$

Note that both $\Delta H$ and $\Delta E$ increased as expected (the temperature is higher).

c) The heat into the system (the gas) for this constant pressure situation is $q = \Delta H = 8.73 \text{ kJ}$. Again, heat must flow in to raise the T and increase the V of the gas – it makes sense.
2.15. Indicate for the following processes whether \( q, w, \Delta E \) and \( \Delta H \) are positive, negative or zero.

(a) Two copper bars, one at 80 °C and other at 20 °C are brought into contact in an insulated container; (b) a sample of liquid in an insulated container is stirred by a mechanical linkage to an external motor; (c) a sample of \( \text{H}_2 \) gas is mixed with an equimolar amount of \( \text{N}_2 \) gas at constant T and P where no chemical reaction occurs.

a) The system is the two bars. Since the system is in a thermally isolated container, no heat flows to or from the system, so \( q = 0 \). The volume of the hot copper bar will decrease and that of the cool bar increase, so the net change will be zero, thus \( w = 0 \). Because \( q \) and \( w \) are zero, \( \Delta E \) is zero. Since \( \Delta E = 0 \) and \( P \Delta V = 0 \), it must be that \( \Delta H = \Delta E + P \Delta V = 0 \).

b) As in a), the system is thermally isolated, so \( q = 0 \). The mechanical linkage allows work (stirring) to be done on the liquid, so \( w > 0 \). \( \Delta E = q + w > 0 \) since \( q = 0 \) and \( w > 0 \). This is a liquid, so \( \Delta(PV) \) is negligibly small. Thus, \( \Delta H = \Delta E + \Delta(PV) > 0 \).

c) The wording suggests that the two gases are ideal. Thus, when they mix there is no T change, and thus \( q = C_p \Delta T = 0 \). There is no volume change (the sum of the volumes does not change), so there is no work done. That means that \( \Delta E = q + w = 0 \). Likewise, it must be that \( \Delta H = \Delta E + P \Delta V = 0 \).
2.20. 100 g of liquid water at 55 °C is mixed with 10 g of ice at -10 °C. P is constant and no heat leaves the system. Calculate the final temperature.

There is much more hot water than cold ice, so it is likely that the ice will melt completely. So let’s assume that and calculate the heat needed to raise the temperature of the ice to 0 °C. This is a simple heat capacity problem:

\[
q_{\text{ice\_warm}} = mC\Delta T = (10g)(2.113kJ/K\cdot kg)(273K-(-263k))(/1kg/1000g) = 0.2113kJ
\]

Removing that much heat from the 55 °C liquid water will change its temperature (another heat capacity problem) by

\[
\Delta T = q_{\text{cool}} / mC = -0.2113kJ / (0.1kg)(4.18kJ/K\cdot kg) = -0.506K.
\]

That is, the new liquid water temperature is lower by 0.506 degrees, or it is 54.494 °C. Note that the heat added to the hot water is negative. It flows into the colder ice to melt it.

So now we have 10 g of ice at 0 °C and 100 g of liquid water at 54.494 °C. The ice will melt, and to do that requires heat for melting, for which we need to use the mass of ice and its latent heat of melting:

\[
q_{\text{melt}} = mL_m = (10g)(334.4kJ/K\cdot kg)(/1kg/1000g) = 3.334kJ
\]

As before, this heat must come from the warm water, so its temperature will drop once again:

\[
\Delta T = q_{\text{cool}} / mC = -3.334kJ / (0.1kg)(4.18kJ/K\cdot kg) = -7.976K
\]

So the temperature of the hot water drops an additional 7.976 degrees to 46.52 °C. At this point we have two masses of liquid water at two different temperatures. They will reach the same final temperature, and to do so they the hot batch will give heat to the cold batch. Thus, we can equate (with a minus sign) the heat capacity heat for each pool of water:

\[
m_{100}C\Delta T_{100} = m_{10}C\Delta T_{10}
\]

or

\[
(100g)C(T_f - 46.52^\circ C) = -(10g)C(T_f - 0^\circ C)
\]

The specific heat capacity cancels on both sides, so we solve for the final T:

\[
(100g)(T_f) + (10g)(T_f) = -(10g)(0^\circ C) + (100g)(46.52^\circ C)
\]

or

\[
T_f = (100g)(46.52^\circ C)/(110g) = 42.29^\circ C. \text{ This makes sense: the final result is fairly close to the original temperature of the much more massive liquid water.}
\]
2.28. Crops can yield 20 kg of sucrose per hectare per hr. in sunlight of 1 kW/m². (a) Using standard enthalpies of formation, calculate the enthalpy of production of sucrose from CO₂(g) and H₂O(l), (b) calculate the energy needed to synthesize 20 kg of sucrose and (c) compute the efficiency of conversion of sunlight to sucrose.

a) Look up the enthalpies of formation of the reaction components from the appendices. Be careful about the phases (liq, gas, solid).

\[
\begin{align*}
CO_2(g): & -393.5 \text{ kJ/mol} \\
H_2O(l): & -285.8 \text{ kJ/mol} \\
Sucrose(s): & -2222 \text{ kJ/mol} \\
O_2(g): & 0 \text{ kJ/mol}
\end{align*}
\]

The enthalpy change in one mole of the reaction is

\[
\Delta H = (1\text{mol})(-2222 \text{kJ/mol}) + (12\text{mol})(-0\text{kJ/mol}) - (1\text{mol})(-285.8\text{kJ/mol}) - (12\text{mol})(-393.5\text{kJ/mol}) = 5644\text{kJ}
\]

Note that the reaction requires energy to proceed.

b) The number of moles of sucrose in 20 kg is \(n = (20 \text{kg})(1000 \text{g/kg})/(342.3 \text{g/mol}) = 58.4 \text{ mol}\), so the total enthalpy change for the production of 58.4 mol of sucrose is \(\Delta H = (58.4 \text{ mol})(5644 \text{kJ/mol}) = 330 \text{ MJ}\). This is the amount produced in one hectare in one hr.

c) In one hectare (10,000 m²), the energy that is available from sunlight in one hour is

\[
E = (1 \text{ kW/m² })(1 \text{ hr})(10^4 \text{ m}^2)(1000 \text{ W/kW})(1 \text{ J/sec·W})(60 \text{ sec/min})(60 \text{ min/hr}) = 3.6 \times 10^{10} \text{ J} = 36,000 \text{ MJ}.
\]

So the efficiency \(e = (\text{energy captured})/(\text{energy input}) = (330 \text{ MJ})/(36,000 \text{ MJ}) = 0.00917\), pretty low efficiency.